1 TDC MTH COMM (S)

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MATHEMATICS (Speciality)

(For Commerce)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. *(a)* *(i)* Sum the series to n terms : 3

$$7+77+777+\cdots$$

(ii) If ${}^{n}P_{r}=132$ and ${}^{n}C_{r}=66$, find n and r. 4

(b) For what value of k, the lines $3x+2y+7=0$ and $2x+ky+9=0$ will be perpendicular to each other? 3

(c) Find the ratio in which the point (4, 1) divides the join of the points (6, − 1) and (3, 2). 3

(d) Express the equation $5x-4y-2=0$ in the intercept form. 1

OR

2. *(a)* The sum of three numbers in AP is 33 and their product is 1287, find the numbers. 4

(b) If $x^{1/a} = y^{1/b} = z^{1/c}$ and $y = \sqrt{zx}$, show that $a + c = 2b$. 2

(c) Find the equation of a straight line which cuts the X-axis and Y-axis in the intercepts a and b respectively. 5

(d) Find the equation of the line passing through the point $A(-2, 4)$ having slope equal to $-4/5$. 3

3. *(a)* *(i)* A function is an odd function of x if $f(-x) =$ ——. (Fill in the blank) 1

(ii) If $f(x) = \dfrac{1}{x}$, prove that

$$f(p) - f(q) = f\left(\frac{pq}{q-p}\right)$$ 3

(iii) State the conditions under which a function is continuous at a given point. 2

(b) (i) Find the value of

$$\lim_{x \to 2} \frac{x^2 + 2x - 2}{2x + 3}$$ 2

(ii) Evaluate : 2

$$\underset{x \to \infty}{\text{Lt}} \frac{2x + 3}{3x + 4}$$

(c) If $y = \frac{x}{\sqrt{x^2 + 1}}$, find $\frac{d^2y}{dx^2}$ 4

OR

4. (a) Evaluate : 2½×2=5

(i) $\int \left(x + \frac{1}{x^2} \right) dx$

(ii) $\int_0^1 \frac{2x}{1 + x^2} dx$

(b) If $z = \frac{x^2 y^2}{x + y}$, prove that

$$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 3z$$ 3

(c) If $x^m y^n = (x + y)^{m + n}$, show that

$$\frac{dy}{dx} = \frac{y}{x}$$ 4

(d) Find $\frac{dy}{dx}$, when $y = x^2 e^x$. 2

5. (a) (i) Under what conditions value of a determinant is zero? 1

(ii) Find the minor and cofactor of 5 in Δ : 2

$$\Delta = \begin{vmatrix} 1 & -2 & 2 \\ 3 & 2 & 5 \\ 1 & -3 & 7 \end{vmatrix}$$

(b) Find the value of

$$\begin{bmatrix} 1 & 3 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} 1 & 0 & 3 \\ 5 & -7 & 2 \end{bmatrix} + \begin{bmatrix} 8 & 9 & 2 \\ 5 & 3 & 9 \end{bmatrix}$$ 3

(c) If $x + y + z = 0$, show that

$$\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^3 \end{vmatrix} = 0$$ 5

(d) Find the inverse of

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$ 3

OR

6. (a) If

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \\ 4 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 2 & 0 \\ 2 & 3 & 0 \end{bmatrix}, \text{ find}$$

BA. Can we find *AB* also? 3+1=4

(b) If

$$A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}, \text{ find } (A - 2I)(A - 3I)$$ 3

(c) If $A = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$ and $k = 5$, verify $(kA)^T = kA^T$. 2

(d) Solve by Cramer's rule : 5

$$2y - 3z = 0$$
$$x + 3y = -4$$
$$3x + 4y = 3$$

7. (a) Define LPP. What are the basic assumptions of LPP? 2+3=5

(b) Discuss the limitation of linear programming. 4

(c) Solve the following LPP using graphical method : 5

Maximize $Z = 3x_1 + 4x_2$

subject to

$$4x_1 + 2x_2 \leq 80$$
$$2x_1 + 5x_2 \leq 180$$
$$x_1 \geq 0$$
$$x_2 \geq 0$$

OR

8. *(a)* Define any *two* of the following : $1\frac{1}{2}\times2=3$

Feasible region; Feasible solution and Slack variable.

(b) Who first developed LPP? Discuss the scope of linear programming in solving business problem. 1+4=5

(c) A firm manufactures two types of products P_1 and P_2 and sells them at a profit of Rs 5 on type P_1 and Rs 4 on type P_2. Each product is processed on two machines M_1 and M_2. Type P_1 requires one minute of processing time on M_1 and two minutes on M_2; type P_2 requires one minute on M_1 and one minute on M_2. The machine M_1 is available for not more than 5 hours 30 minutes while machine M_2 is available for 8 hours and 20 minutes during any working day. Formulate the problem as a linear programming problem. 6

9. *(a)* *(i)* Given $\log 23\cdot31=1\cdot3676$,

find antilog $(-2\cdot6324)$. 2

(ii) If $\log_{0\cdot01} x=-1$, find x. 1

(b) (i) Define Sinking Fund. 2

(ii) A person borrowed Rs 6,000 from a moneylender but he could not pay any amount in a period of 4 years. Accordingly the moneylender demanded now Rs 7,500 from him. What rate percent per annum compound interest did the latter require for lending his money? 4

(c) A person buys a television paying Rs 5,000 cash and promising to pay Rs 400 at the end of every month for the next four years. If money is worth 12% p.a., compounded monthly, what is the cash price of the television? 5

OR

10. (a) If $\log \frac{x+y}{7} = \frac{1}{2}(\log x + \log y)$, find the value of $\left(\frac{x}{y} + \frac{y}{x}\right)$ 3

(b) A certain sum of money invested on compound interest amounts to Rs 2,420 in 2 years and Rs 2,662 in 3 years. Find the rate of interest and the sum invested. 4

(c) Define perpetuity and deferred annuity. 2

(d) A man retires at the age of 60 and his employer gives him a pension of Rs 1,200 a year paid in half-yearly instalments for the rest of his life. Assuming his expectation of life to be 13 years and that interest is 4% p.a. payable half-yearly, what will be the present value of his total amount of pension? 5

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